AE3-410 Mathematics

Course Aims

- To introduce the basic concepts of variational calculus and their applications
- To introduce the theory of complex variables and techniques based on complex variables

Syllabus

Calculus of variations: Euler-Lagrange equations; problems with constraints.
Functions of a complex variable: Revision of complex numbers: triangle inequality, polar coordinate representation, curves in the complex plane. Continuity and differentiability of complex functions: analyticity, the Cauchy-Riemann equations. Definitions and properties of elementary complex functions. Branches and branch points. Complex line integrals: definition and properties. Cauchy's integral theorem and its consequences. Cauchy's integral formula. Complex power series: Taylor series, Laurent series. Classification of singularities in the complex plane: poles, residues and essential singularities. The residue theorem: contour integration, evaluation of real integrals.
Laplace transforms: Revision of basic properties. Derivation of complex inversion formula. Use of contour integration. Application to differential equations.
Conformal Mapping: Application to Laplace's equation. The Joukowski transformation.

Pre-Requisites AE1-107 Mathematics AE2-209 Mathematics

Learning Outcomes

Knowledge and understanding

On successfully completing this course unit, students will be able to:

- Formulate and solve the Euler-Lagrange equations;
- Apply the basic theory of functions of a complex variable;
- Derive Cauchy's theorem and evaluate the integral of a simple complex function around a curve in the complex plane;
- Derive power series expansions of complex functions about singular points of the functions;
- Distinguish between the different types of singularities that can arise in the complex plane;
- Derive Cauchy's residue theorem and use it to evaluate real integrals over finite and infinite ranges;
- Calculate inverse Laplace transforms using the residue theorem and apply this technique to solve certain partial differential equations;
- Use the ideas of conformal mappings to solve equations in geometrically complicated domains (e.g. to determine the inviscid flow field around a Joukowski airfoil).

Skills and other attributes

On successfully completing this course unit, students will have acquired the following skills:
Intellectual skills

- The ability to think clearly and pay attention to detail;
- The ability to manipulate expressions algebraically, and minimize the making of errors.

Practical skills

- *Problem-solving: the ability to formulate a problem precisely and then solve it logically, making all assumptions clear.*
- *Investigative skills: e.g. researching material on-line and in the library, asking others for advice.*

Transferable skills

- Communication skills – to be successful you will need to develop your listening and note-taking skills.
- Determination – to solve the most difficult problems you will need to persevere.
- Creativity – some problems require a combination of techniques.
- Intellectual rigour.
- Ability to work independently.

Teaching Methods

Lectures, tutorials. Teaching via printed material and whiteboard.

Assessment
Examined Assessment
2 hour written closed-book examination in the Summer term (100%)
Non-Examined Assessment

Course Analysis:

	TOPIC	NO. OF LECTURES
(i)	Calculus of variations	3
(ii)	Complex variables	6
(iii)	Complex power series	3
(iv)	Residue theorem; contour integration	2
(v)	Laplace Transform	3
(iv)	Conformal mappings	3

Reading List
Category as defined by Central Library:
Core – Multiple copies available; Supplementary – 1 or 2 copies available.

Title: Advanced Mathematical Methods for Engineering and Science Students
Author: Stephenson G., Radmore P. M.
Publisher: Cambridge University Press
Category: Supplementary

Title: Advanced Engineering Mathematics
Author: Wylie C. R., Barrett L. C.
Publisher: McGraw-Hill
Category: Supplementary

Title: Advanced Engineering Mathematics
Author: Kreyszig E.
Publisher: Wiley
Category: Supplementary

Title: Complex Variables: Introduction and Application
Author: Ablowitz M. J., Fokas, A. S.
Publisher: Cambridge University Press
Category: Supplementary

Additional References:

Mathematical methods in the physical sciences, by Mary L Boas (2nd Edition or 3rd Edition): Chap 9 (Calculus of variations), Chap 14 (Complex variables and conformal mapping) and Chap 15 (Laplace transforms)

Advanced modern engineering mathematics by Glyn James: Chap 8 (Complex variables and conformal mapping) and Chap 9 (Laplace transforms)

Introduction to complex analysis by H. A Priestley. The materials about complex variables are explained in greater details in this book (you can skip Chap 2,3,9,15 16 and 22)